ACHARYA INSTITUTE OF TECHNOLOGY
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10MAT31

Third Semester B.E. Degree Examination, Dec.2016/Jan.2017
Engineering Mathematics – III

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Obtain the Fourier series in $(-\pi, \pi)$ for $f(x) = x\cos x$. **(07 Marks)**

b. Obtain the Fourier half range sine series,

$$f(x) = \begin{cases} \frac{1}{4} - x & \text{in } 0 < x < \frac{1}{2} \\ x - \frac{3}{4} & \text{in } \frac{1}{2} < x < 1 \end{cases}.$$
(07 Marks)

c. Obtain the constant term and the coefficients of the first cosine and sine terms in the Fourier expansion of y from the table. **(06 Marks)**

x	0	1	2	3	4	5
y	9	18	24	28	26	20

2 a. Find the Fourier transforms of $f(x) = \begin{cases} 1 - x^2 & \text{for } |x| < 1 \\ 0 & \text{for } |x| \geq 1 \end{cases}$ and hence evaluate

$$\int_0^\infty \frac{x\cos x - \sin x}{x^3} \cos\frac{x}{2}\, dx.$$
(07 Marks)

b. Find the Fourier sine transform of $e^{-|x|}$. **(07 Marks)**

c. Find the inverse Fourier sine transform of $\hat{f}_s(\alpha) = \frac{e^{-a\alpha}}{\alpha}$, $a > 0$. **(06 Marks)**

3 a. Solve the wave equation $u_{tt} = c^2 u_{xx}$ given that $u(0,t) = 0 = u(2l, t)$, $u(x, 0) = 0$ and $\frac{\partial u}{\partial t}(x,0) = a\sin^3\frac{\pi x}{2l}$ **(07 Marks)**

b. Solve the boundary value problem $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ $0 < x < l$, $\frac{\partial u}{\partial x}(0,t) = 0$, $\frac{\partial u}{\partial x}(l,t) = 0$, $u(x, 0) = x$. **(07 Marks)**

c. Obtain the D'Almbert's solution of the wave equation, $u_{tt} = C^2 u_{xx}$ subject to the conditions $u(x,0) = f(x)$ and $\frac{\partial u}{\partial t}(x,0) = 0$. **(06 Marks)**

4 a. Fit a parabola $y = a + bx + cx^2$ for the data: **(07 Marks)**

x	0	1	2	3	4
y	1	1.8	1.3	2.5	2.3

b. Solve by using graphical method the L.P.P.
Minimize $z = 30x + 20y$
Subject to the constraints: $x - y \leq 1$
$x + y \geq 3$, $y \leq 4$
and $x \geq 0$, $y \geq 0$ **(07 Marks)**

c. Maximize $z = 3x + 4y$
subject to the constraints $2x + y \leq 40$, $2x + 5y \leq 180$,
$x \geq 0, y \geq 0$ using simplex method. **(06 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – B

5 a. Find the fourth root of 12 correct to three decimal places by using regula Falsi method. **(07 Marks)**

b. Solve $9x-2y+z=50$, $x+5y-3z=18$, $-2x+2y+7z=19$ by relaxation method obtaining the solution correct to two decimal places. **(07 Marks)**

c. Find the largest eigen value and the corresponding eigen vector of, $\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ by using power method by taking initial vector as $[1 \quad 1 \quad 1]^T$. **(06 Marks)**

6 a. The table gives the values of $\tan x$ for $0.10 \le x \le 0.30$ **(07 Marks)** using extrapolation formula find tan(0.26)

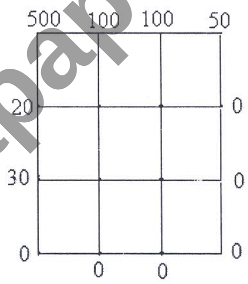
x	0.10	0.15	0.20	0.25	0.30
tanx	0.1003	0.1511	0.2027	0.2553	0.3093

b. Using Newton's forward and backward interpolation formula, calculate the increase in population from the year 1955 to 1985. The population in a town is given by, **(07 Marks)**

Year	1951	1961	1971	1981	1991
Population in thousands	19.96	39.65	58.81	77.21	94.61

c. Evaluate $\int_0^1 \frac{dx}{1+x}$ taking seven ordinates by applying Simpson's $\frac{3}{8}^{th}$ rule. Hence deduce the value of $\log_e 2$. **(06 Marks)**

7 a. Solve the Laplace's equation $u_{xx}+u_{yy}=0$, given that **(07 Marks)**



b. Solve $\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2}$ subject to $u(0,t)=0$; $u(4,t)=0$; $u(x,0)=x(4-x)$. Take h = 1, K = 0.5 upto Four steps. **(07 Marks)**

c. Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ subject to the condition u(x, 0) = $\sin \pi x$, $0 \le x \le 1$, $u(0,t)=u(1,t)=0$ using Schmidt's method. Carry out computations for two levels, taking $h=\frac{1}{3}$, $K=\frac{1}{36}$. **(06 Marks)**

8 a. Find the z-transform of, (i) $\cosh n\theta$ (ii) $\sinh n\theta$ **(07 Marks)**

b. Obtain the inverse z-transform of, $\frac{4z^2-2z}{z^3-5z^2+8z-4}$. **(07 Marks)**

c. Solve the difference equation,
$y_{n+2}+2y_{n+1}+y_n = n$ with $y_0 = y_1 = 0$ using z-transforms. **(06 Marks)**

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